SIEVE of ERATOSTHENES

Use the Sieve of Eratosthenes to investigate primes, composites, multiples, and prime factorizations.

Who was Eratosthenes?
Eratosthenes, a Greek mathematician, invented the “sieve method” for finding prime #'s about 22 000 years ago. This activity explores a variation of Eratosthenes’ sieve.

Getting Started…
- **One is neither prime nor composite.** To show this, mark an X through the ‘1’ box.
- The first prime number is 2. Colour the diamond in which 2 is located ORANGE.
- Use RED to colour the upper-left corner of the key and upper-left corner of all squares containing multiples of 2 (skip count by 2 OR multiples of 2).
  - The numbers with just a corner coloured “fell through the sieve”!

  .... What was the first multiple of 2 that “fell through” the sieve? Answer: _______

- The next prime number is 3. Colour the diamond surrounding the 3 ORANGE.
- Use BLUE to colour the upper-right corner of the key and the upper-right corner of all squares containing multiples of 3 (skip count by 3 OR multiples of 3).
- Repeat this process for prime numbers 5 and 7. Colour the diamonds surrounding these numbers ORANGE.
- Use GREEN to colour the lower-right corners of the key and of the squares containing multiples of 5 (skip count by 5 OR multiples of 5).
- Use YELLOW to colour the lower-left corners of the key and of the squares containing multiples of 7 (skip count by 7 OR multiples of 7).

  .... What are the first multiples of 5 that fell through the sieve? _______ And 7? _______

- Finally, use ORANGE to colour the diamond surrounding all the numbers in the grid that are in squares with no corners coloured. These numbers are all PRIME NUMBERS!

Use this key to show the correct colour-coding.
Application & Reasoning
Using your completed colour-coded sieve, respond to the following questions in detail to demonstrate your full understanding.

1. How do you know that 2, 3, 5, and 7 are prime numbers?

2. How can you tell this from the way the sieve is coloured?

3. How can you identify composite numbers from the way the sieve is coloured?

Remember….
When you coloured the multiples of 2, the number 4 was the first multiple of 2 that “fell through the sieve”. When you coloured multiples of 3, the number 9 was the first multiple of 3 that fell through the sieve.

4. What was the first multiple of 3 that fell through? Why wasn’t 6 the first multiple of 3 to fall through?

5. When colouring multiples, what was….
   a) The first multiple of 5 that fell through the sieve? How do you know?
   b) The first multiple of 7 that fell through the sieve? How do you know?

   … After colouring the multiples of 7, the next uncoloured number was 11.

6. When you colour multiples of 11, what is the first number that will fall through the sieve?

7. When you colour multiples of any prime number, how is the first multiple that falls through the sieve related to that prime number?

8. If the grid went to 300, what is the largest prime number you would reach (whose multiples must be coloured) before you can be certain that all of the remaining uncoloured numbers are prime?

9. What is the largest prime less than 100? Explain your answer.
Sieve

1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30
31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100